the only base you really need: e

1) exponential functions

$$y = a^x = (e^{\ln a})^x = e^{(\ln a)x}$$

always convert to the natural exponential function with the identity $\boxed{a = e^{\ln a}}$.

2) logarithmic functions

$y = \log_a x$ means $x = a^y$

"y is the exponent you need to raise a to to get x"

$$\ln x = \ln a^y = y \ln a$$

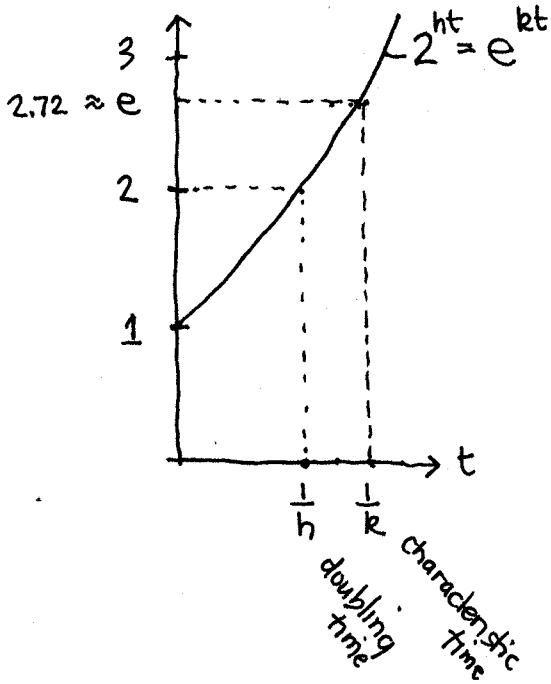
$$y = \frac{\ln x}{\ln a}$$

$$\boxed{\log_a x = \frac{\ln x}{\ln a}}$$

use this identity to convert to natural log

3) base 2: doubling times, half life

population growth or radioactive decay can be described with either base 2 or base e



same growth curve, different growth rates with respect to base 2 and base 3

doubling time: $1/h$

characteristic time: $1/k$

$$\ln[2^{ht} = e^{kt}]$$

$$\ln(2^{ht}) = \ln(e^{kt})$$

$$ht(\ln 2) = kt$$

$$k = (\ln 2)h \approx .693h$$

$$\frac{1}{k} = \left(\frac{1}{\ln 2}\right)\frac{1}{h} \approx 1.44\frac{1}{h}$$

takes longer to grow by factor of $e \approx 2.72$ than a factor of 2