L'Hôpital's Rule and indeterminate limits of power form

Graph $y = x^x$ ← both base and exponent variable so can be rewritten as an exponential

$= (e^{\ln x})^x = e^{x \ln x}$

domain: $x > 0$ since base of an exponent expression should be positive to avoid problems

1st derivative info $\quad \frac{dy}{dx} = \frac{d}{dx} e^{x\ln x} = e^{x \ln x} \underbrace{\frac{d}{dx}(x \ln x)}_{1 \ln x + x\left(\frac{1}{x}\right)} = x^x (1 + \ln x) = 0 \rightarrow \ln x = -1$

$x = e^{\ln x} = e^{-1} \approx .368$

$y = (e^{-1})^{e^{-1}} = \left(\frac{1}{e}\right)^{1/e} = \frac{1}{e^{1/e}} \approx .692$

y': $-$ 0 $+$

e^{-1}

(since $\ln x$ is increasing, $1 + \ln x$ crosses thru zero from neg to pos values)

2nd derivative info $\quad \frac{d^2y}{dx^2} = \frac{d}{dx}\left[e^{x \ln x}(1 + \ln x)\right] = \underbrace{\left(\frac{d}{dx} e^{x \ln x}\right)}_{e^{x\ln x}(1+\ln x) \text{ (above)}} (1 + \ln x) + e^{x \ln x}\left(0 + \frac{1}{x}\right)$

$= e^{x \ln x}\left[(1 + \ln x)^2 + \frac{1}{x}\right] > 0$ → concave up (since $x > 0$)

limits at edge of domain: $\lim_{x \to \infty} x^x = \infty^\infty = \infty$ (determinate limit)

$\lim_{x \to 0^+} x^x \sim 0^0$ indeterminate since $0^x = 0$ for $x > 0$

but $x^0 = 1$ for $x > 0$

competition between base and exponent

limit evaluation

$y = x^x$

$\ln y = \ln x^x = x \ln x$

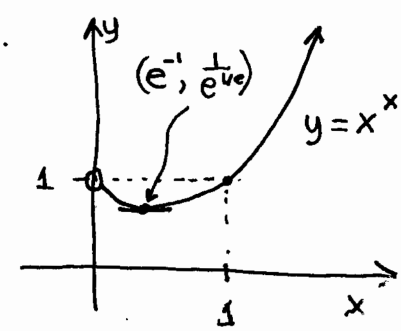
$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} x \ln x \sim 0 \cdot \ln 0^+ = 0 \cdot (-\infty)$ indeterminate product

$= \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} \sim \frac{-\infty}{\infty}$ indeterminate quotient, L'Hôpital's Rule applies

$= \lim_{x \to 0^+} \frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}\left(\frac{1}{x}\right)} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{\left(-\frac{1}{x^2}\right)} = \lim_{x \to 0^+} (-x) = 0$

$\lim_{x \to 0^+} y = \lim_{x \to 0^+} e^{\ln y} = e^{\lim_{x \to 0^+} \ln y} = e^0 = \underline{1}.$

no problem rule e^x continuous: limit passes thru



similarly ∞^0 and 0^∞ are other types of symbolic indeterminate limits that can be handled this way