Rules of (explicit) differentiation

constants rules

c constant function rule $\frac{d}{dx} c = 0$

$\pm c$ additive constant rule $\frac{d}{dx}(f(x) \pm c) = \frac{d}{dx} f(x)$

$*c$, $/c$ constant factor rule: multiplier $\frac{d}{dx}(c\, f(x)) = c\, \frac{d}{dx} f(x)$

divisor $\frac{d}{dx}\left(\frac{f(x)}{c}\right) = \frac{\frac{d}{dx} f(x)}{c}$

constant numerator rule:

$$\frac{d}{dx}\left(\frac{c}{f(x)}\right) = \frac{d}{dx}\, c(f(x))^{-1} = c(-1)\, f(x)^{-2} \frac{d}{dx} f(x)$$

chain rule
constant factor rule
power rule

operation rules

$+/-$ sum/difference $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

$*$ product $\frac{d}{dx}(f(x) g(x)) = \frac{d}{dx} f(x) \cdot g(x) + f(x) \frac{d}{dx} g(x)$

$\div$ quotient $\frac{d}{dx}\left(\frac{top(x)}{bot(x)}\right) = \frac{bot(x) \frac{d}{dx} top(x) - top(x) \frac{d}{dx} bot(x)}{bot(x)^2}$

$\circ$ composition $\rightarrow$ chain rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \frac{d}{dx} g(x) \quad , \quad \frac{d}{dx} f(u) = f'(u) \frac{du}{dx}$$

short list of functions

$\frac{d}{dx} x = 1$ identity

$\frac{d}{dx} x^p = p x^{p-1}$ power

$\frac{d}{dx} e^x = e^x$ exp

$\frac{d}{dx} \ln x = \frac{1}{x}$ "log"

$\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} \cos x = -\sin x$

"arctrig" $\uparrow$

other "trig" $\leftarrow$ look up

with chain rule:

$$\frac{d}{dx} u^p = p u^{p-1} \frac{du}{dx} \ , \ \frac{d}{dx} e^u = e^u \frac{du}{dx} \ , \ \frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx} \ , \ \frac{d}{dx}(\text{trig})(u) = \text{trig}'(u) \frac{du}{dx}$$

All of these rules are useless if you cannot use them in context, for example, if $PV = C$ (constant) so that $P = C/V$ or $V = C/P$, can you evaluate dP/dV or dV/dP?