FUNCTIONS OF 1 INDEPENDENT VARIABLE

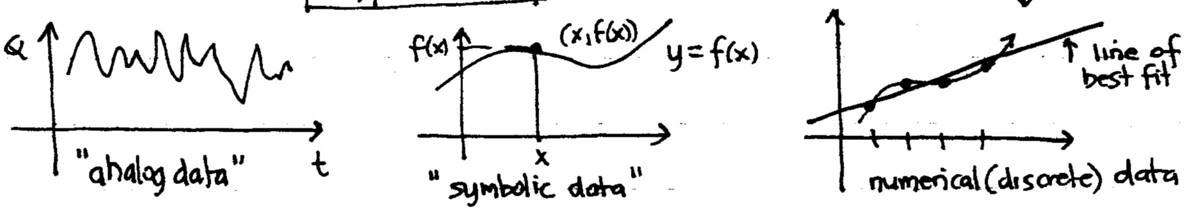
REPRESENTATIONS:

1) symbolic $f(x) =$ formula + domain

2) numerical discrete data: (table)

x	1	2	4	5
f(x)	2	3	3	4

3) graphical



4) verbal *The square function produces the square of the input number as its output.*

inverse functions:

functional relationship $y = f(x) \xrightarrow[\text{for } x]{\text{solve}} x = f^{-1}(y)$ [both represent same curve in x-y plane]

inverse function graph $y = f^{-1}(x) \leftrightarrow x = f(y)$ ← if switch x & y in functional relationship get inverse functional relationship curve.

geometric operations on graphs:

operation			on argument	on function value
translation	+C	c > 0 c < 0	← left → right	↑ up ↓ down
positive scaling	*C	c > 1 c < 1	→← compression ←→ expansion	←→ expansion →← compression
reflection	*(−1)			
			even, if unchanged	odd if unchanged under both at once

reflection symmetry:

$$f(-x) = \begin{cases} f(x) & \text{even} \\ -f(x) & \text{odd} \end{cases}$$

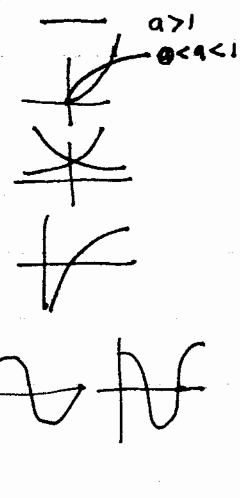
Building up functions:

particular functions

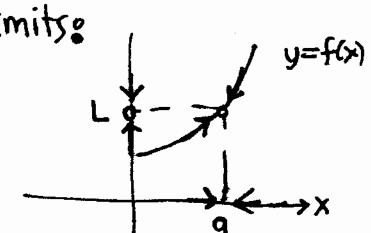
constants	c
powers	x^a
exp's	e^x
log's	$\ln x$
trig (inv trig)	$\cos x, \sin x, \ldots$ ...

+ operations on functions: $+/-$, $*$, $\div$, $\circ$ → expressions for functions we can handle in calculus. polynomials, rational functions, etc.

graphs



limits:



graphical

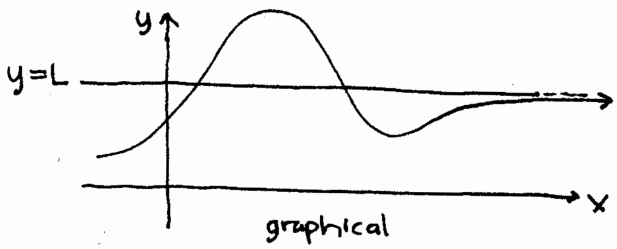
f(1.1)
f(1.01)
f(1.001)
...
numerical (zoom in)

symbolic: [factor & cancel, expand & cancel, sqrt conjugation] $\frac{0}{0}$

later: [L'Hopital's rule $\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty, 1^\infty, \infty^0, 0^0, \infty-\infty$]

rat'ls (divide by largest denom power)

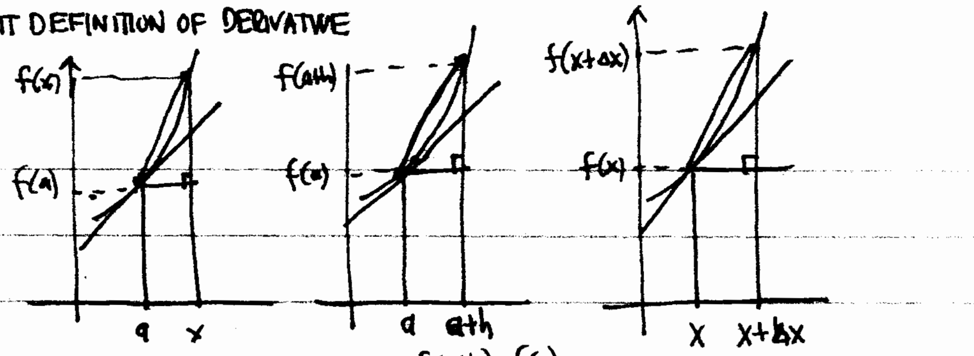
limits at $\pm\infty$



$f(10.)$
$f(100.)$
$f(1000.)$
$f(10{,}000.)$
...

numerical (zoom out)

LIMIT DEFINITION OF DERIVATIVE



$f'(a) = \lim\limits_{x\to a} \dfrac{f(x)-f(a)}{x-a} = \lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$ $\quad f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x)-f(x)}{\Delta x}$

interpretation of derivative of functional relationship

$y = f(x)$

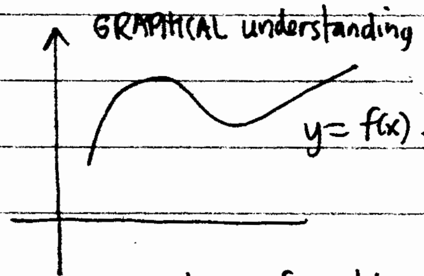
$\downarrow$

$\dfrac{dy}{dx} = \lim\limits_{\Delta x\to 0} \underbrace{\dfrac{\Delta y}{\Delta x}}_{\text{avg rate of change}}$ instant. rate of change.

$\downarrow$

[extend derivative to tabular data]

GRAPHICAL understanding



$y = f(x) \to$ derivative $y = f'(x)$
second derivative $y = f''(x)$

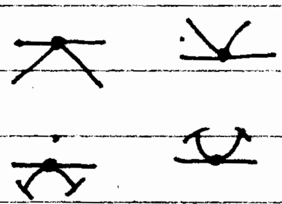
Derivative information about graph

$\boxed{\text{sgn } f'}$ + inc
− dec
0 hor tanline
∞ vert tanline
cusp
jump kink

} crit pts

$\boxed{\text{sgn } f''}$ + concave up
− concave down
switch sign inflection

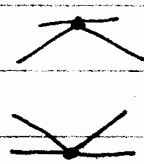
$\boxed{\text{local max/min}}$



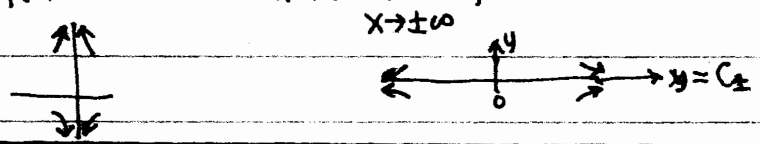
1st der test

2nd der test.

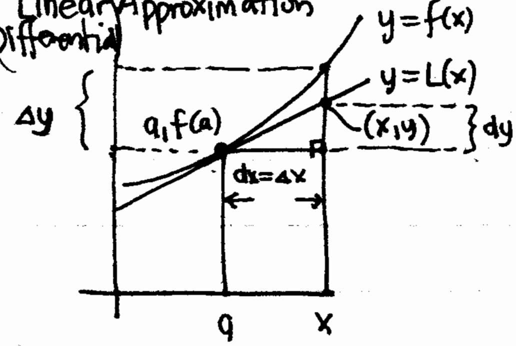
abs max/min on interval



ASYMPTOTES vert: $\lim\limits_{x\to a^\pm} f(x) = \pm\infty$ hor: $\lim\limits_{x\to\pm\infty} f(x) = C_\pm$



(Linear Approximation / Differential)



$\frac{y-f(a)}{x-a}=f'(a) \rightarrow y=f(a)+f'(a)(x-a)=L(x)$

$\frac{dy}{dx}=f'(a) \rightarrow dy=f'(a)\,dx$

or

$dy = f'(x)\,dx$

$dy\big|_{x=a} = f'(a)\,dx$

$dy\big|_{\substack{x=a\\dx=\Delta x}} = f'(a)\,\Delta x$

[$\frac{dy}{y}=\frac{f'(x)}{f(x)}dx$
relative change (approximate)
2% change: $\frac{dx}{x}=.02$]

Derivative rules

function rules: $c,\ x^a,\ e^x,\ \ln x,\ \cos x,\ \sin x, \ldots$

operation rules: $+/-,\ *,\ \div,\ \circ$

Differentiation

explicit $\frac{d}{dx}[f(x)] = \cdots = f'(x)$

[logarithmic differentiation] prods, quots, powers $\rightarrow$ $y=f(x)$
$\ln y = \ln f(x) = \cdots$ expand.
$\rightarrow \ldots$

implicit $\frac{d}{dx}[F(x,y)=0] \rightarrow \cdots$ solve for $\frac{dy}{dx}=G(x,y)$

[inverse functions $y=f^{-1}(x) \Leftrightarrow x=f(y) \rightarrow \cdots \Rightarrow \frac{dy}{dx}=\frac{d}{dx}f^{-1}(x)$]

related rates $\frac{d}{dx}[F(y,z)] = \cdots$ (chain rule differentiation)
$\rightarrow$ solve for unknown rate if all other values known

Max/min word problems $F(x,y)=0$ relationship, eliminate y

quantity to extremize $Q=G(x,y)=G(x,\text{"}y(x)\text{"})=\text{"}Q(x)\text{"}$ + domain $\rightarrow$ local/max/min. etc.