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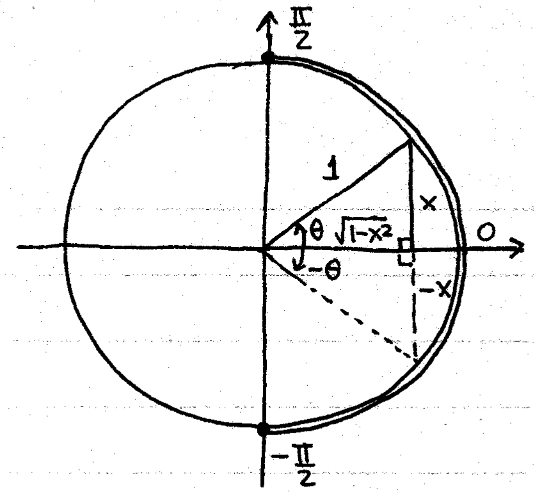


$x = \sin\theta \qquad \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\theta = \underbrace{\sin^{-1} x}$

read this mentally as "the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is x"

$\sin^{-1}$ takes values "on the righthand side of the unit circle

This is the picture assuming $x > 0$ (first quadrant).
If $x < 0$, the ref. Δ is in the 4^{th} quadrant.

Note that $x \to -x$ means $\theta \to -\theta$ so both $\sin$ and $\sin^{-1}$ are odd functions

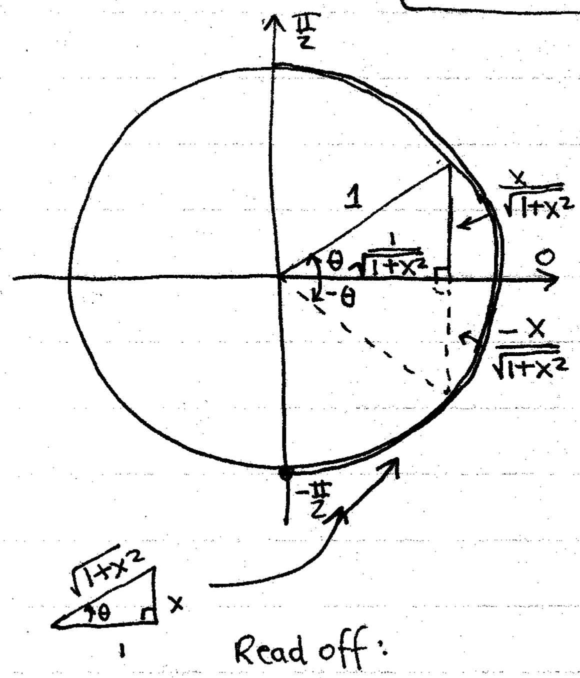
Read off the other trig functions from the reference Δ:

$\cos\theta = \cos(\sin^{-1}x) = \sqrt{1-x^2}$, $\sec\theta = \sec(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$, [both positive in these quadrants]

$\tan\theta = \frac{x}{\sqrt{1-x^2}} = \tan(\sin^{-1}x)$, $\cot\theta = \cot(\sin^{-1}x) = \frac{\sqrt{1-x^2}}{x}$, $\csc\theta = \csc(\sin^{-1}x)$ [same sign as x in these quadrants]

$$\boxed{y = \sin^{-1}x \to x = \sin y \quad \frac{dx}{dy} = \cos y \quad \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\cos(\sin^{-1}x)} = \frac{1}{\sqrt{1-x^2}}}$$

$$\boxed{\frac{d}{dx}[\sin^{-1}x] = \frac{1}{\sqrt{1-x^2}}}$$



$x = \tan\theta \qquad \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\theta = \underbrace{\tan^{-1} x}$

read this mentally as "the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is x."

$\tan^{-1}$ takes values "on the righthand side of the unit circle.

This is the picture assuming $x > 0$ (first quadrant)
If $x < 0$, the ref Δ is in the 4^{th} quadrant.

Note that $x \to -x$ means $\theta \to -\theta$ so both $\tan$ and $\tan^{-1}$ are odd functions

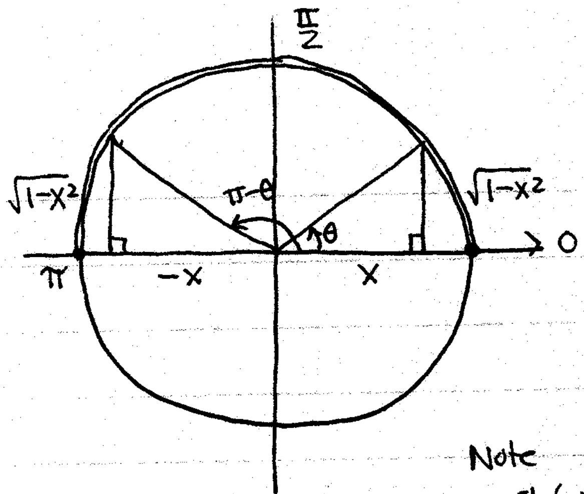
Read off:

$\cos\theta = \cos(\tan^{-1}x) = \frac{1}{\sqrt{1+x^2}}$, $\sec\theta = \sec(\tan^{-1}x) = \sqrt{1+x^2}$ [both positive in these quadrants]

$\sin\theta = \sin(\tan^{-1}x) = \frac{x}{\sqrt{1+x^2}}$, $\csc\theta = \csc(\tan^{-1}x) = \frac{\sqrt{1+x^2}}{x}$, $\cot\theta = \cot(\tan^{-1}x) = \frac{1}{x}$ [same sign as x in these quadrants]

$$\boxed{y = \tan^{-1}x \quad x = \tan y \quad \frac{dx}{dy} = \sec^2 y \cdot \quad \frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{\sec^2(\tan^{-1}x)} = \frac{1}{(1+x^2)}}$$

$$\boxed{\frac{d}{dx}[\tan^{-1}x] = \frac{1}{(1+x^2)}}$$



$X = \cos\theta \qquad \theta \in [0,\pi]$

$\theta = \cos^{-1}x$

read this mentally as "the angle between 0 and π whose cosine is x"

$\cos^{-1}$ takes values "on the upper half of the unit circle."

This is the picture assuming $x>0$ (first quadrant).

If $x<0$ the ref Δ is ~~the~~ the 2nd quadrant

Note $\quad x \to -x$ means $\theta \to \pi-\theta$, i.e.

$\cos^{-1}(-x) = \pi-\theta = \pi - \cos^{-1}x.$

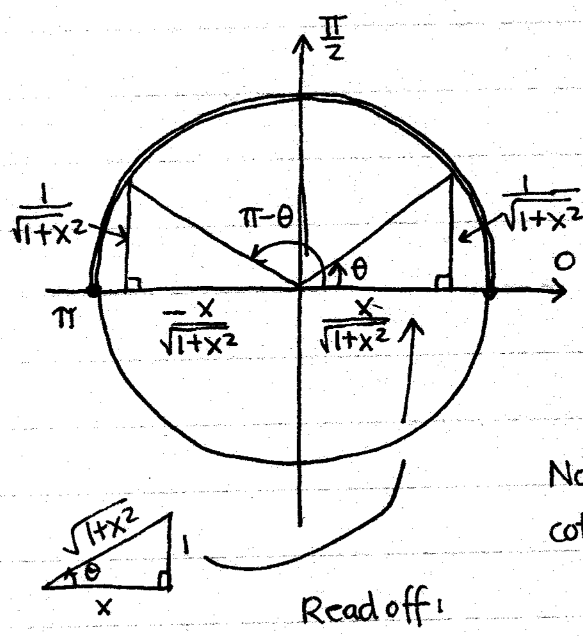
Read off:

$\sin\theta = \sin(\cos^{-1}x) = \sqrt{1-x^2}, \quad \csc\theta = \csc(\cos^{-1}x) = \frac{1}{\sqrt{1-x^2}}$ [both positive in these quadrants]

$\tan\theta = \tan(\cos^{-1}x) = \frac{\sqrt{1-x^2}}{x}, \quad \cot\theta = \cot(\cos^{-1}x) = \frac{x}{\sqrt{1-x^2}}, \quad \sec\theta = \sec[\cos^{-1}x] = \frac{1}{x}$ [same sign as x in these quadrants]

$$\boxed{y = \cos^{-1}x \quad x = \cos y \quad \frac{dx}{dy} = -\sin y \quad \frac{dy}{dx} = \frac{-1}{\sin y} = \frac{-1}{\sin(\cos^{-1}x)} = \frac{-1}{\sqrt{1-x^2}}}$$

$$\boxed{\frac{d}{dx}[\cos^{-1}x] = \frac{-1}{\sqrt{1-x^2}}}$$



$x = \cot\theta \qquad \theta \in [0,\pi]$

$\theta = \cot^{-1}x$

read this mentally as "the angle between 0 and π whose cotangent is x".

$\cot^{-1}$ takes values on the "upper half of the unit circle

This is the picture assuming $x>0$ (first quadrant)

If $x<0$, ref Δ is in the 2nd quadrant.

Note $\quad x \to -x$ means $\theta \to \pi-\theta$, i.e.

$\cot^{-1}(-x) = \pi-\theta = \pi - \cot^{-1}x.$

Read off:

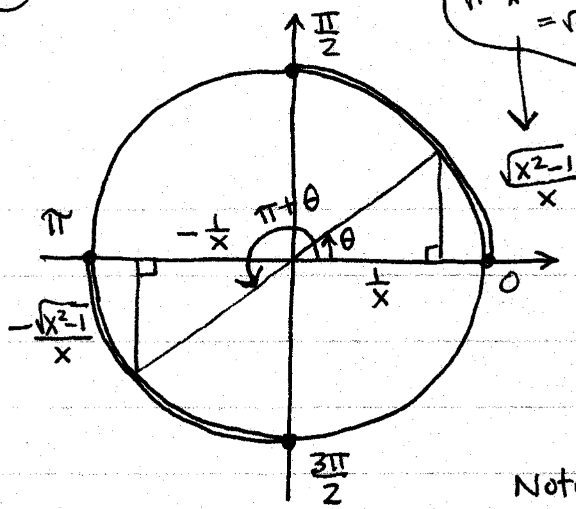
$\sin\theta = \sin(\cot^{-1}x) = \frac{1}{\sqrt{1+x^2}}, \quad \csc\theta = \csc(\cot^{-1}x) = \sqrt{1+x^2}$ [both positive...]

$\cos\theta = \cos(\cot^{-1}x) = \frac{x}{\sqrt{1+x^2}}, \quad \sec\theta = \sec(\cot^{-1}x) = \frac{\sqrt{1+x^2}}{x}, \quad \tan\theta = \tan(\cot^{-1}x) = \frac{1}{x}$ [same sign as x...]

$$\boxed{y = \cot^{-1}x \quad x = \cot y \quad \frac{dx}{dy} = -\csc^2 y \quad \frac{dy}{dx} = -\frac{1}{\csc^2 y} = \frac{-1}{\csc^2(\cot^{-1}x)} = \frac{-1}{(1+x^2)}}$$

$$\boxed{\frac{d}{dx}[\cot^{-1}x] = \frac{-1}{(1+x^2)}}$$

$$\sqrt{1-\tfrac{1}{x^2}} = \sqrt{\tfrac{x^2-1}{x^2}} = \tfrac{\sqrt{x^2-1}}{x}$$



$x = \sec\theta \qquad \theta \in [0, \frac{\pi}{2}] \cup [\pi, \frac{3\pi}{2}]$

$\theta = \underbrace{\sec^{-1} x}$

read this mentally as "the angle between 0 and $\frac{\pi}{2}$ or π and $\frac{3\pi}{2}$ whose secant is x

$\sec^{-1}$ takes values " in the 1st and 3rd quadrants" of the unit circle".

This is the picture assuming $x > 0$ (first quadrant)

If $x < 0$, ref △ is in 3rd quadrant.

Note that $x \to -x$ means $\theta \to \pi + \theta$, i.e.

$\sec^{-1}(-x) = \pi + \theta = \pi + \sec^{-1} x$.

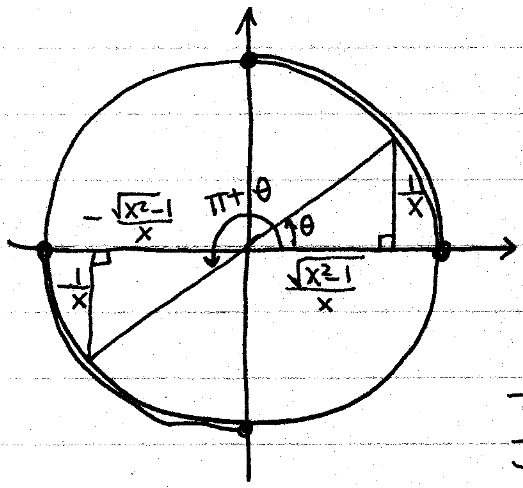
Read off:

$\cos\theta = \cos(\sec^{-1}x) = \frac{1}{x}$, $\sin\theta = \sin(\sec^{-1}x) = \frac{\sqrt{x^2-1}}{x}$, $\csc\theta = \csc(\sec^{-1}x) = \frac{x}{\sqrt{x^2-1}}$ (same sign as x...)

$\tan\theta = \tan(\sec^{-1}x) = \sqrt{x^2-1}$, $\cot\theta = \cot(\sec^{-1}x) = \frac{1}{\sqrt{x^2-1}}$ (both positive...)

$y = \sec^{-1}x \quad x = \sec y \quad \frac{dx}{dy} = \sec y \tan y \quad \frac{dy}{dx} = \frac{1}{\sec y \tan y} = \frac{1}{x \tan(\sec^{-1}x)} = \frac{1}{x\sqrt{x^2-1}}$

$$\frac{d}{dx}[\sec^{-1}x] = \frac{1}{x\sqrt{x^2-1}}$$



$x = \csc\theta$

$\theta = \underbrace{\csc^{-1} x}$

read this mentally as " the angle between 0 and $\frac{\pi}{2}$ or π and $\frac{3\pi}{2}$ whose cosecant is x."

$\csc^{-1}$ takes values " in the 1st and 3rd quadrants" of the unit circle.

This is the picture assuming $x > 0$ (1st quadrant)

If $x < 0$, ref △ is in 3rd quadrant.

Note that $x \to -x$ means $\theta \to \pi + \theta$, i.e.

$\csc^{-1}(-x) = \pi + \theta = \pi + \csc^{-1}(x)$.

$\sin\theta = \sin(\csc^{-1}x) = \frac{1}{x}$, $\cos\theta = \cos(\csc^{-1}x) = \frac{\sqrt{x^2-1}}{x}$, $\sec\theta = \sec(\csc^{-1}x) = \frac{x}{\sqrt{x^2-1}}$ [same sign as x ...]

$\cot\theta = \cot(\csc^{-1}x) = \sqrt{x^2-1}$, $\tan\theta = \tan(\csc^{-1}x) = \frac{1}{\sqrt{x^2-1}}$ [both positive...]

$y = \csc^{-1}x \quad x = \csc y \quad \frac{dx}{dy} = -\csc y \cot y \quad \frac{dy}{dx} = \frac{-1}{\csc y \cot y} = \frac{-1}{x\cot(\csc^{-1}x)} = \frac{-1}{x\sqrt{x^2-1}}$

$$\frac{d}{dx}[\csc^{-1}(x)] = -\frac{1}{x\sqrt{x^2-1}}$$

Remember!