Some Algebra Rules

distributive rule: $a(b+c) \underset{\xleftarrow{\text{factor}}}{\overset{\xrightarrow{\text{expand}}}{=}} ab + ac$ $\quad [a+ac = a\cdot 1 + ac = a(1+c)]$

fractions:

$\frac{a}{b}+\frac{c}{d} = \frac{ad}{bd}+\frac{bc}{bd} = \frac{ad+bc}{bd}$ add numerators if same denominator

$\frac{a}{b}+c = \frac{a}{b}+\frac{bc}{b} = \frac{a+bc}{b}$ (cannot cancel b's in last expression)

$a^{-1} = \frac{1}{a}$, $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

$\frac{c}{\left(\frac{a}{b}\right)} = \left(\frac{b}{a}\right)c = \frac{bc}{a}$ $\qquad \frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$

[always factor before cancelling: $\frac{a+ac}{ab} = \frac{\cancel{a}^{1}(1+c)}{\cancel{a}_{1}b} = \frac{1+c}{b}$]

cancellation:

$*$: $\frac{a\cancel{b}^{1}}{\cancel{b}_{1}} = a$

$+$: $a + \underbrace{\cancel{b} - \cancel{b}}_{0} = a$

	exponentials $(a>0)$:	logs $(\ln = \log_e)$:
product	$a^x a^y = a^{x+y}$	$\ln xy = \ln x + \ln y$
quotient	$a^x/a^y = a^{x-y}$	$\ln x/y = \ln x - \ln y$
power	$(a^x)^y = a^{xy}$	$\ln x^p = p\ln x$
reciprocal	$(a^x)^{-1} = a^{-x} = 1/a^x$	$\ln \frac{1}{x} = \ln x^{-1} = -\ln x$

only base e needed:

$a = e^{\ln a}$

$a^x = (e^{\ln a})^x = e^{(\ln a)x}$

$\log_a x = \frac{\ln x}{\ln a}$

exp/ln properties: $e^{\ln x} = x \ (x>0)$ $\quad e^0 = 1$ $\quad e^1 = e \approx 2.718$

$\ln e^x = x$ $\quad \ln 1 = 0$

$[\, e^{p\ln x} = (e^{\ln x})^p = x^p \,]$ $\quad \frac{1}{e^x} = e^{-x}$ (preferred)

powers/roots:

product	$(xy)^p = x^p y^p$	$x^0 = 1 \quad (x \neq 0)$
quotient	$(x/y)^p = x^p/y^p$	$1^x = 1$
power	$(x^p)^q = x^{pq}$	$\sqrt{x^2} = \lvert x\rvert = \begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$
reciprocal	$x^{-p} = 1/x^p$	

$x^{\frac{1}{n}} = \sqrt[n]{x}$, n integer >0

always convert radical notation to power notation to work with in calculus

$x^{\frac{m}{n}} = (x^{\frac{1}{n}})^m = (\sqrt[n]{x})^m$

$= (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$

$(a+b)^2 \underset{\xleftarrow{\text{factor}}}{\overset{\xrightarrow{\text{expand}}}{=}} a^2 + 2ab + b^2$ (*binomial theorem for higher integer powers*)

Some Algebra Rules 2

solving equations

a) linear: $ax+b=0 \rightarrow x=-b/a \quad (a\neq 0)$

b) quadratic: $ax^2+bx+c=0 \rightarrow x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a} \quad (a\neq 0)$ memorize!

related technique: completing the square

$$ax^2+bx+c = a\underbrace{\left(x^2+\tfrac{b}{a}x\right)}_{\left(x+\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2} + c \qquad \left[\text{since} = x^2+\cancel{2}x\left(\tfrac{b}{\cancel{2}a}\right)+\left(\tfrac{b}{2a}\right)^2 \cancel{-\left(\tfrac{b}{2a}\right)^2}\right]$$

$$= a\left(x+\tfrac{b}{2a}\right)^2 - a\left(\tfrac{b^2}{4a^2}\right)+c$$

$$= a\left(x+\tfrac{b}{2a}\right)^2 + \left(c-\tfrac{b^2}{4a}\right)$$

dont memorize formula
remember technique

Distinguish between "solving an equation" for an unknown variable which appears in it, as opposed to "simplifying or rewriting" an expression, as in:

$x^2-2x = x(x-2)$ (factored)

Note: $f(x)\,g(x)=0 \longrightarrow$ either $f(x)=0$ or $g(x)=0$

$\dfrac{f(x)}{g(x)}=0 \longrightarrow f(x)=0$, provided that $g(x)\neq 0$ at a solution of $f(x)=0$

rules of algebra NOT!

$\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$, $\quad \sqrt{a^2+b^2}\neq a+b$

$\dfrac{c}{a}+\dfrac{c}{b} \neq \dfrac{c}{a+b} \qquad \dfrac{ab+c}{ad} \neq \dfrac{b+c}{d}$

$\sqrt{x^2}\neq x$ unless x is known to be positive ($|x|$!)

$f(2x)\neq 2f(x)$ (unless f is very special!)

$\sin 2x \neq 2\sin x$

cancellation NOT!

$\times$: $\dfrac{a+\cancel{b}c}{\cancel{b}} \neq a+c$

$+$: $\cancel{b}+a(c-\cancel{b}) \neq ac$

Miscellaneous

$0x=0$ (says nothing about x)

$\dfrac{0}{x}=0$ when $x\neq 0$

$\dfrac{x}{0}$ never defined

math is case sensitive: $T\neq t$, $R\neq r$

even roots of negative #'s are complex, not real:

$\sqrt[2n]{x}$, $x<0$ is undefined over real #'s for $n>0$ integer